Error localization

A bounded constraint-based approach to aid for error localization

Bekkouche Mohammed, Collavizza Hélène, Rueher Michel

Univ. Nice Sophia Antipolis, CNRS, I3S, UMR 7271
06900 Sophia Antipolis, France
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Plan

Introduction

Motivation

- BMC(Bounded Model Checking) and testing tools can generate one or more counterexamples
- A counterexample provides an execution trace
- **The trace of the counterexample** is often long and complicated to understand
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Inputs

- A program contradicts its specification : the violated postcondition POST
- A counterexample CE provided by a BMC tool

Outputs

A reduced set of suspicious statements allowing the programmer to understand the **origin of his mistakes**

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1. The program is modeled in a CFG in DSA form
2. The program and its specification are translated in numerical constraints
3. CE : a counterexample, PATH : **an erroneous path**
4. The CSP $C = CE \cup PATH \cup POST$ **is inconsistent**

Key points

- What are the erroneous instructions on $PATH$ that make C **inconsistent** ?
- Which subsets remove to make C **feasible** ?
- What **paths** to explore ? $\rightarrow$ path of CE, **deviations** from CE

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Approach

MCS: Minimal Correction Subset

MCS: Definition

Let C an **infeasible** set of constraints

$$M \subseteq C \text{ is a } \mathbf{MCS} \Leftrightarrow \begin{cases} M \subseteq C \\ Sol(< X, C \backslash M, D >) \neq \emptyset \\ \nexists C'' \subset M : Sol(< X, C \backslash C'', D >) = \emptyset \end{cases}$$

MCS: Example

- $C = \{c_1 : i = 0, c_2 : v = 5, c_3 : w = 6, c_4 : z = i + v + w, c_5 : ((z = 0 \vee i \neq 0) \wedge (v \geq 0) \wedge (w \geq 0))\}$ **is inconsistent**
- C has 4 **MCS**: $\{c_1\}$, $\{c_4\}$, $\{c_5\}$,$\{c_2, c_3\}$

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(LocFaults) algorithm

- Isolation of **MCS** on the **path** of CE
- DFS exploration of CFG by propagating CE and **by deviating** at most k conditional statements $c_1, .., c_k$
  - P: propagation constraints derived from CE (of the form $variable = constant$)
  - C: constraints of **chemin** up to c_k
  - If $\underline{P \models POST}$:
    * $\{\neg c_1, .., \neg c_k\}$ is a **correction**,
    * **MCS** of $C \cup \{\neg c_1, .., \neg c_k\}$ are **corrections**
- A bound for the **MCS** calculated and the conditions **deviated**

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Example

Calculate the absolute value of i-j

```
class AbsMinus {
/*returns|i-j|,the absolute value of i minus j*/
/*@ ensures
  @ (result==|i-j|);
  @*/
  int AbsMinus (int i, int j) {
    int result;
    int k = 0;
    if (i <= j) {
       k = k+2;    // error : k = k+2 instead of
              k=k+1
    }
    if (k == 1 && i != j) {
        result = j-i;
    }
    else {
        result = i-j;
    }
    return result;
  }
}
```

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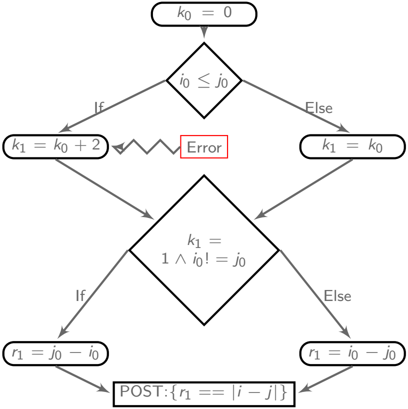


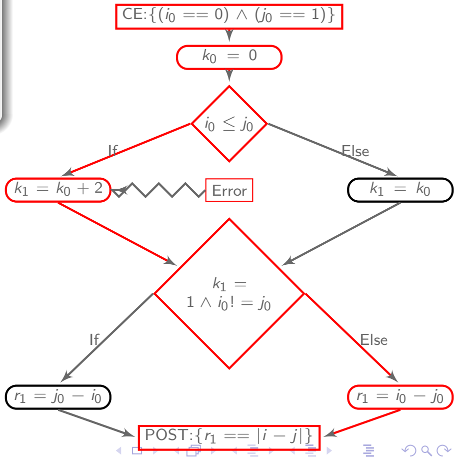
Example

The **path** of the counterexample

POST:$\{r_1 == |i - j|\}$

$\{i_0 = 0, j_0 = 1, k_0 = 0, k_1 = k_0 + 2, r_1 = i_0 - j_0, r_1 = |i - j|\}$ **is inconsistent**

Only one MCS on the path : $\{r_1 = i_0 - j_0\}$



Exemple

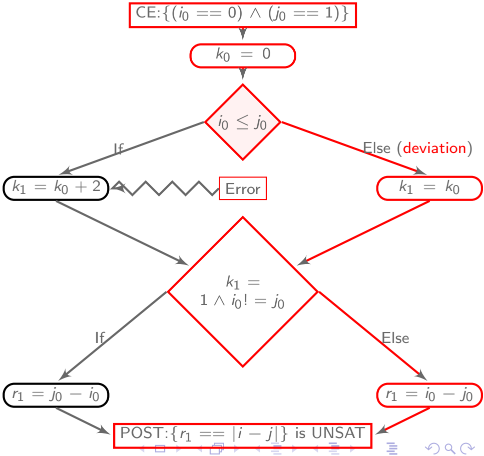
The **path** obtained **by deviating** the condition $i_0 \leq j_0$

The **deviated** condition : $\{i_0 \leq j_0\}$

$P = \{i_0 = 0, j_0 = 1, k_0 = 0, k_1 = 0, r_1 = -1\}$

$P \cup \{r_1 = |i - j|\}$ **is inconsistent**

The deviation $\{i_0 \leq j_0\}$ **does not correct the program**



Example

The **path by deviating** the condition $k_1 = 1 \wedge i_0! = j_0$

The **deviated** condition : $\{(k_1 = 1 \wedge i_0! = j_0)\}$

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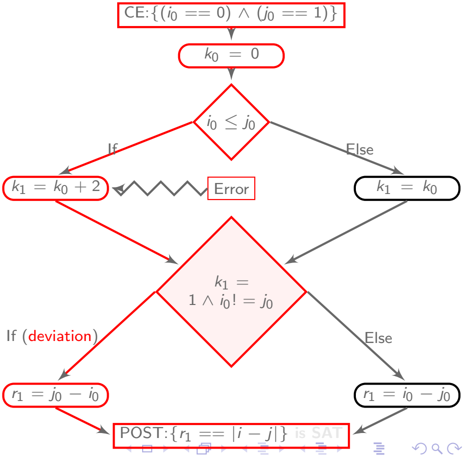
The deviation $\{(k_1 = 1 \wedge i_0! = j_0)\}$ **corrects** the program

$C = \{i_0 = 0, j_0 = 1, k_0 = 0, k_1 = k_0 + 2, \neg(k_1 = 1 \wedge i_0! = j_0)\}$

C **is inconsistent**

MCS on the path : $\{k_0 = 0\}, \{k_1 = k_0 + 2\}$



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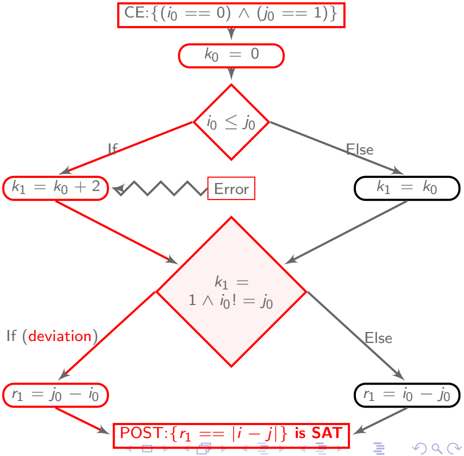

State of the art

Approaches based on systematic testing

Tarantula, Delta Debugging

- Ranking of suspicious statements detected during the execution of a test battery

+ Simple approaches
– Need **many** test cases
Approaches that require the existence of an oracle
→ Decide if the result of **tens of thousands** of test is **just**

Our framework is less demanding
→ Bounded Model Checking

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SAT-based approaches

BugAssist

- A BMC method, like ours
- **Major differences :**
  - → It transforms the entire program into a SAT formula
  - → It based on the use of MaxSAT solvers

+ Global approach
− The complement of the MaxSAT set does not necessarily correspond to the instructions on the same path
  → Displaying the union of these complements

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Experimental evaluation

Tools used

- **LocFaults**: our implementation
  → The MIP solver of IBM ILOG CPLEX
  → The tool CPBPV to generate the CFG and CE
  → Benchmarks: Java programs
- **BugAssist**: the tool of error localization for `BugAssist` approach
  → The MaxSAT solver MSUnCore2
  → Benchmarks: ANSI-C programs

Experimental evaluation

Programs built

- **Simple programs**
  - → AbsMinus, Minmax, Mid (illustrative programs)
  - → Maxmin6var (program without junctions)
  - → Tritype, TriPerimetre (programs with junctions)
  - → Several erroneous versions for each program

    Example: TriPerimetre → TriPerimetreKO, TriPerimetreKO2, TriPerimetreKO3
- **TCAS(Traffic Collision Avoidance System), a realistic benchmark**
  - → 1608 test cases, except cases for overflow *PositiveRAAltThresh* table
  - → TcasKO . . . TcasKO41

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Results (**MCS** identified)

Program	Counterexample	LocFaults = 0	LocFaults ≤ 1	LocFaults ≤ 2	LocFaults ≤ 3	BugAssist
AbsMinusKO3	$\{i = 0, j = 1\}$	{20}	{16},{14},{12} {20}	{16},{14},{12} {20}	{16},{14},{12} {20}	{16, 20}
MinmaxKO	$\{in_1 = 2, in_2 = 1, in_3 = 3\}$	{10},{19}	{18},{10} {10},{19}	{18},{10} {10},{19}	{18},{10} {10},{19}	{18, 19, 22}
MidKO	$\{a = 2, b = 1, c = 3\}$	{19}	{19}	{19}	{14, 23, 26} {19}	{14, 19, 30}
Maxmin6varKO4	$\{a = 1, b = -3, c = -4, d = -2, e = -1, f = -2\}$	{116}	{116}	{116}	{12, 15, 19} {116}	{12, 166}
TritypeKO2	$\{i = 2, j = 2, k = 4\}$	{54}	{21} {26} {35},{27},{25} {53},{25},{27} {54}	{21} {26} {29, 57},{30},{27},{25} {32, 44},{33},{25},{27} {35},{27},{25} {53},{25},{27} {54}	{21} {26} {29, 57},{30},{27},{25} {32, 44},{33},{25},{27} {35},{27},{25} {53},{25},{27} {54}	{21, 26, 27, 29, 30, 32, 33, 35, 36, 53, 68}
TritypeKO4	$\{i = 2, j = 3, k = 3\}$	{46}	{45},{33},{25} {46}	{26, 32} {29, 32} {45},{33},{25} {46}	{26, 32} {29, 32} {32, 35, 49},{25} {32, 35, 53},{25} {32, 35, 57},{25} {45},{33},{25} {46}	{26, 27, 29, 30, 32, 33, 35, 45, 49, 68}
TriPerimetreKO	$\{i = 2, j = 1, k = 2\}$	{58}	{31} {37},{32},{27} {58}	{31} {37},{32},{27} {58}	{31} {37},{32},{27} {58}	{28, 29, 31, 32, 35, 37, 65, 72}

LocFaults provides a more informative and explanatory localization

Experimental evaluation

Results (time calculation)

Program	LocFaults					BugAssist	
	P	L				P	L
		$=0$	≤ 1	≤ 2	≤ 3		
AbsMinusKO3	$0,479s$	$0,076s$	$0,113s$	$0,357s$	$0,336s$	$0,02s$	$0,04s$
MinmaxKO	$0,528s$	$0,243s$	$0,318s$	$0,965s$	$1,016s$	$0,01s$	$0,09s$
MidKO	$0,524s$	$0,065s$	$0,078s$	$0,052s$	$0,329s$	$0,02s$	$0,08s$
Maxmin6varKO4	$0,538s$	$0,06s$	$0,07s$	$0,075s$	$0,56s$	$0,04s$	$0,78s$
TritypeKO2	$0,51s$	$0,023s$	$0,25s$	$2,083s$	$3,864s$	$0,02s$	$0,69s$
TritypeKO4	$0,497s$	$0,023s$	$0,095s$	$0,295s$	$5,127s$	$0,02s$	$0,21s$
TriPerimetreKO	$0,518s$	$0,047s$	$0,126s$	$1,096s$	$2,389s$	$0,03s$	$0,64s$

The times of `LocFaults` are close to the times of `BugAssist`

Experimental evaluation

Results (Number of errors localized for TCAS)

Programme	Nb_E	Nb_CE	LF ≤ 1	LF ≤ 2	BA
TcasKO	1	131	131	131	131
TcasKO2	2	67	67	134	67
TcasKO3	1	23	2	2	23
TcasKO4	1	20	16	20	20
TcasKO5	1	10	10	10	10
TcasKO6	3	12	36	36	24
TcasKO7	1	36	23	36	0
TcasKO8	1	1	1	1	0
TcasKO9	1	7	7	7	7
TcasKO10	6	14	16	65	84
TcasKO11	6	14	16	34	46
TcasKO12	1	70	52	52	70
TcasKO13	1	4	3	3	4
TcasKO14	1	50	6	6	51
TcasKO16	1	70	22	70	0
TcasKO17	1	35	22	35	0
TcasKO18	1	29	21	28	0
TcasKO19	1	19	13	19	0
TcasKO20	1	18	18	18	18
TcasKO21	1	16	16	16	16
TcasKO22	1	11	11	11	11
TcasKO23	1	41	41	41	41
TcasKO24	1	7	7	7	7
TcasKO25	1	3	0	3	3
TcasKO26	1	11	11	11	11
TcasKO27	1	10	10	10	10
TcasKO28	2	75	74	148	121
TcasKO29	2	18	17	35	0
TcasKO30	2	57	57	114	0
TcasKO34	1	77	77	77	77
TcasKO35	4	75	74	148	115
TcasKO36	1	122	120	120	0
TcasKO37	4	94	110	235	236
TcasKO39	1	3	0	3	3
TcasKO40	2	122	0	103	120
TcasKO41	1	20	17	20	20

The performances of `LocFaults` are favorably comparable to `BugAssist`

Conclusion

- Bounded approach for error localization
  - **Bounded DFS** exploration
  - **Bounded MCS** calculation

    → To prevent the combinatorial explosion
- Based on the use of constraint solvers
- `LocFaults` provides more accurate and relevant results compared to `BugAssist`

  → The times of the two tools are similar
- `LocFaults` locates errors frequently for the TCAS programs

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- Incremental version of the algorithm
- Treat programs with loops
- Extend our implementation to support more complex non-linear instructions through the use of **specialized solvers**
- Experiment more of real programs
- Treat the programs with floating-point numbers computation

  $\rightarrow$ Use **specialized solvers** on floating computations

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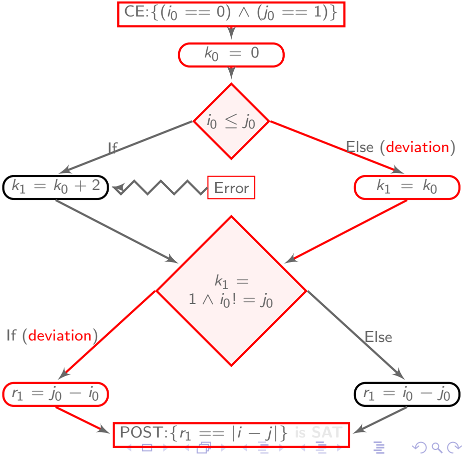
Thank you for your attention.

Questions ?

Example

The **path of a non-minimal deviation** : $\{i_0 \leq j_0, k_1 = 1 \wedge i_0 != j_0\}$

The **deviated** conditions :
$\{i_0 \leq j_0, (k_1 = 1 \wedge i_0 != j_0)\}$
$P = \{i_0 = 0, j_0 = 1, k_0 = 0, k_1 = 0, r_1 = 1\}$
$P \cup \{r_1 = |i - j|\}$ **is consistent**
The deviation is not minimal



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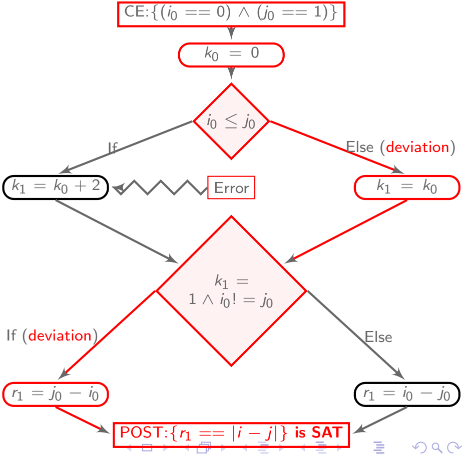

Approach

The algorithm of Liffiton and Sakallah to compute the MCS subsets

Function MCS(C,MCS_b)
Inputs C: Infeasible set of constraints, MCS_b: integer
Outputs MCS: List of MCS in C of a cardinality less than MCS_b
$C' \leftarrow \textsc{AddYVars}(C)$; $MCS \leftarrow \emptyset$; $k \leftarrow 1$;
while $\underline{\mathrm{SAT}(C') \land k \leq MCS_b}$ **do**
  $C'_k \leftarrow C' \land \textsc{AtMost}(\{\neg y_1, \neg y_2, \ldots, \neg y_n\}, k)$
  while $\underline{\mathrm{SAT}(C'_k)}$ **do**
    $newMCS \leftarrow \emptyset$
    forall the $\underline{\text{indicator } y_i}$ **do**
      Let y_i indicator of the constraint $c_i \in C$, and $val(y_i)$ the value of y_i in the solution calculated of C'_k.
      si $\underline{val(y_i) = 0}$ **alors**
        $newMCS \leftarrow newMCS \cup \{c_i\}$.
      fin
    end
    $MCS.add(newMCS)$.
    $C'_k \leftarrow C'_k \land \textsc{BlockingClause}(newMCS)$
    $C' \leftarrow C' \land \textsc{BlockingClause}(newMCS)$
  end
  $k \leftarrow k + 1$
end

return MCS

Approach

The algorithm of Liffiton and Sakallah to compute the MCS subsets

Function MCS(C,MCS_b)
Inputs C: Infeasible set of constraints, MCS_b: integer
Outputs MCS: List of MCS in C of a cardinality less than MCS_b
$C' \leftarrow$ AddYVars(C); $MCS \leftarrow \emptyset$; $k \leftarrow 1$;
while $\underline{\mathrm{SAT}(C') \wedge k \leq MCS_b}$ **do**
  $C'_k \leftarrow C' \wedge$ AtMost($\{\neg y_1, \neg y_2, \ldots, \neg y_n\}$,$k$)
  while $\underline{\mathrm{SAT}(C'_k)}$ **do**
    $newMCS \leftarrow \emptyset$
    forall the $\underline{\text{indicator } y_i}$ **do**
      Let y_i indicator of the constraint $c_i \in C$, and $val(y_i)$ the value of y_i in the solution calculated of C'_k.
      si $\underline{val(y_i) = 0}$ **alors**
        $newMCS \leftarrow newMCS \cup \{c_i\}$.
      fin
    end
    $MCS.add(newMCS)$.
    $C'_k \leftarrow C'_k \wedge$ BlockingClause($newMCS$)
    $C' \leftarrow C' \wedge$ BlockingClause($newMCS$)
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    while $\underline{\mathrm{SAT}(C'_k)}$ **do**
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            fin
        end
        $MCS.add(newMCS)$.
        $C'_k \leftarrow C'_k \wedge \textsc{BlockingClause}(newMCS)$
        $C' \leftarrow C' \wedge \textsc{BlockingClause}(newMCS)$
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  $C'_k \leftarrow C' \wedge \textsc{AtMost}(\{\neg y_1, \neg y_2, \ldots, \neg y_n\}, k)$

  while $\underline{\mathrm{SAT}(C'_k)}$ **do**

    $newMCS \leftarrow \emptyset$

    forall the indicator y_i **do**

      Let y_i indicator of the constraint $c_i \in C$, and $val(y_i)$ the value of y_i in the solution calculated of C'_k.

      si $\underline{val(y_i) = 0}$ **alors**

        $newMCS \leftarrow newMCS \cup \{c_i\}$.

      fin

    end

    $MCS.add(newMCS)$.

    $C'_k \leftarrow C'_k \wedge \textsc{BlockingClause}(newMCS)$

    $C' \leftarrow C' \wedge \textsc{BlockingClause}(newMCS)$

  end

  $k \leftarrow k + 1$

end

return MCS

Approach

The algorithm of Liffiton and Sakallah to compute the MCS subsets

```
Function MCS(C, MCS_b)
Inputs C: Infeasible set of constraints, MCS_b: integer
Outputs MCS: List of MCS in C of a cardinality less than MCS_b
C' ← AddYVars(C); MCS ← ∅; k ← 1;
while SAT(C') ∧ k ≤ MCS_b do
    C'_k ← C' ∧ AtMost({¬y_1, ¬y_2, ..., ¬y_n}, k)
    while SAT(C'_k) do
        newMCS ← ∅
        forall the indicator y_i do
            Let y_i indicator of the constraint c_i ∈ C, and val(y_i) the value of y_i in the solution
            calculated of C'_k.
            si val(y_i) = 0 alors
                newMCS ← newMCS ∪ {c_i}.
            fin
        end
        MCS.add(newMCS).
        C'_k ← C'_k ∧ BlockingClause(newMCS)
        C' ← C' ∧ BlockingClause(newMCS)
    end
    k ← k + 1
end

return MCS
```

Approach

The algorithm of Liffiton and Sakallah to compute the MCS subsets

Function MCS(C,MCS_b)
Inputs C: Infeasible set of constraints, MCS_b: integer
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$C' \leftarrow \textsc{AddYVars}(C)$; $MCS \leftarrow \emptyset$; $k \leftarrow 1$;
while $\text{SAT}(C') \land k \leq MCS_b$ **do**
 $C'_k \leftarrow C' \land \textsc{AtMost}(\{\neg y_1, \neg y_2, \ldots, \neg y_n\}, k)$
 while $\text{SAT}(C'_k)$ **do**
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